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Representation of the virtual Compton amplitude for polarized scattering in the generalized Bjorken region*

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The Compton amplitude is calculated in terms of expectation values of light–ray quark operators. As a technical tool we apply the nonlocal light–cone expansion. Thereby we express the expectation value of the vector light–ray operator with the help of the expectation value of the corresponding scalar operator of twist 2. This allows important simplifications. In the limit of forward scattering the integral relations between the twist–2 contributions of the structure functions are implied directly.

The Compton amplitude for the scattering of a virtual photon off a hadron $\gamma^* + p_1 \rightarrow \gamma^{'*} + p_2$ provides one of the basic tools to understand the short–distance behavior of the nucleon and to test Quantum Chromodynamics (QCD) at large space–like virtualities [1]. The Compton amplitude for the general case of nonforward scattering is given by

$$T_{\mu\nu}(p_+, p_-, q) = i \int d^4x\, e^{iqx} \quad (1)$$
$$\langle p_2, S_2 \left| T(J_\mu(x/2) J_\nu(-x/2)) \right| p_1, S_1 \rangle \ .$$

Here, $p_+ = p_2 + p_1$, $q = (q_1 + q_2)/2, p_- = p_2 - p_1$, where $q_1(q_2)$ and $p_1(p_2)$ denote the four–momenta of the incoming (outgoing) photon and hadron, respectively, and S_1, S_2 are the spins of the initial– and final–state hadron. The *generalized Bjorken region* is defined by the conditions $\nu = qp_+ \rightarrow \infty, Q^2 = -q^2 \rightarrow \infty$, keeping the variables $\xi = Q^2/qp_+$ and $\eta = qp_-/qp_+ = (q_1^2 - q_2^2)/2\nu$ fixed. The (renormalized) time–ordered product in Eq. (1) can be represented in terms of the operator product expansion. The non–local operator product expansion [1–3] leads to compact expressions for the coefficient functions and the operators in the nonforward case. In lowest order in the coupling constant the expansion contains only quark operators with two external legs. The electromagnetic current reads $J_\mu(x) = \overline{\psi}(x)\gamma_\mu \lambda^{\rm em}\psi(x)$, where $\psi(x)$ are the quark fields and $\lambda^{\rm em}$ projects onto the flavor states. One obtains:

$$T(J_\mu(x/2)J_\nu(-x/2)) \approx \int_{-1}^{+1} d\kappa_+ \int_{-1}^{+1} d\kappa_- \quad (2)$$
$$\times \Big[C_a(x^2, \kappa_\pm, \mu^2) \Big(g_{\mu\nu} O^a(\kappa_\pm \tilde{x} \mu^2)$$
$$-\tilde{x}_\mu O^a_\nu(\kappa_\pm \tilde{x}, \mu^2) - \tilde{x}_\nu O^a_\mu(\kappa_\pm \tilde{x}, \mu^2) \Big)$$
$$+ i\, C_{a,5}(x^2, \kappa_\pm, \mu^2) \varepsilon_{\mu\nu}{}^{\rho\sigma} \tilde{x}_\rho O^a_{5,\sigma}(\kappa_\pm \tilde{x}, \mu^2) \Big].$$

For convenience we introduced auxiliary integrations over the variables $\kappa_\pm$, and a light–ray vector $\tilde{x}$ corresponding to the vector x. Here $C_{a(5)}$ are the renormalized coefficient functions which are used in Born approximation, and $O^a_{(5)\mu}(\kappa_1\tilde{x}, \kappa_2\tilde{x})$ are the renormalized (anti)symmetric light–cone operators with $\kappa_\pm = (\kappa_2 \pm \kappa_1)/2$,

$$O^a_\mu = \frac{1}{2}[\overline{\psi}(\kappa_1\tilde{x})\lambda^a_f \gamma_\mu \psi(\kappa_2\tilde{x}) - (\kappa_1 \leftrightarrow \kappa_2)],$$
$$O^a_{5,\mu} = \frac{1}{2}[\overline{\psi}(\kappa_1\tilde{x})\lambda^a_f \gamma_5 \gamma_\mu \psi(\kappa_2\tilde{x}) + (\kappa_1 \leftrightarrow \kappa_2)],$$

respectively. The phase factors drop out in the light–cone gauge $\tilde{x}A = 0$. We consider first the quark operators O^a_μ and $O^a_{5,\mu}$, which contain contributions of twist–2, 3 and 4. The explicit computations lead to the following expressions for the

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twist–2 light–ray vector operators [4] in terms of the corresponding scalar operator

$$O_\mu^{q,\,\mathrm{tw2}}(-\kappa\tilde{x},\kappa\tilde{x}) = \int_0^1 d\tau \partial_\mu\, O^q_{\mathrm{trl}}(-\kappa\tau x,\kappa\tau x)\big|_{x\to\tilde{x}}\,.$$

Let us first consider the matrix elements of the the scalar twist–2 quark operator

$$\langle p_2|O^q_{\mathrm{trl}}(-\kappa_- x,\kappa_- x)|p_1\rangle = \tilde{g}^q(\kappa_- xp_\pm,\kappa_-^2 x^2)\overline{u}(p_2)(\gamma x)u(p_1) + \tilde{h}^q(\kappa_- xp_\pm,\kappa_-^2 x^2)\overline{u}(p_2)x(\sigma p_-)u(p_1)/M. \tag{3}$$

Using the foregoing relation we get an expression for the matrix element of the vector operator,

$$\begin{aligned}
&\langle p_2|O_\mu^{q,\,\mathrm{tw2}}(\kappa_1\tilde{x},\kappa_2\tilde{x})|p_1\rangle \\
&= \int_0^1 d\lambda\,\partial^x_\mu\,\langle p_2|O^q_{\mathrm{trl}}(\kappa_1\lambda x,\kappa_2\lambda x)|p_1\rangle\Big|_{x=\tilde{x}} \\
&= \int_0^1 d\lambda\,\partial^x_\mu\Big\{e^{i\kappa_+\lambda x p_-}\Big[\overline{u}(p_2)(\gamma x)u(p_1) \\
&\quad\times\tilde{g}^q(\kappa_-\lambda xp_+,\kappa_-\lambda xp_-,\kappa_-^2\lambda^2x^2) \\
&\quad+\overline{u}(p_2)x(\sigma p_-)u(p_1)/M \\
&\quad\times\tilde{h}^q(\kappa_-\lambda xp_+,\kappa_-\lambda xp_-,\kappa_-^2\lambda^2x^2)\Big]\Big\}\Big|_{x=\tilde{x}}.
\end{aligned}\tag{4}$$

We now apply the Fourier transformation to $\tilde{g}(\tilde{h})$ with measure $Dz = dz_+dz_-\ \ \theta(1+z_++z_-)$ $\theta(1+z_+-z_-)\theta(1-z_++z_-)\theta(1-z_+-z_-)$:

$$\tilde{f}(\kappa xp_+,\kappa xp_-,\kappa^2x^2) = \int Dz e^{-i\kappa x(p_+z_++p_-z_-)} f(z_+,z_-,\kappa^2x^2)\ , \tag{5}$$

and perform the λ–integration in Eq. (4), which yields

$$\begin{aligned}
&\langle p_2|O_\mu^{q,\,\mathrm{twist2}}(\kappa_1\tilde{x},\kappa_2\tilde{x})|p_1\rangle \\
&= \int Dz e^{-i\kappa_-\tilde{x}(p_+z_++p_-z_-}\Big\{G^q(z_+,z_-) \\
&\quad(\overline{u}(p_2)\gamma_\mu u(p_1) - i\kappa_- p_\mu(z)\overline{u}(p_2)(\gamma x)u(p_1)) \\
&\quad+\overline{u}\gamma_\mu u[(i\kappa_+p_-^\mu\partial_{i\kappa_+xp_-} + 2x_\mu\kappa_-^2\partial_{\kappa_-^2x^2})] \\
&\quad\times G^q(z_+,z_-,\kappa_+xp_-,\kappa_-^2x^2)\Big\}\Big|_{x=\tilde{x}} \\
&\quad+\text{similar terms containing } H^q\ .
\end{aligned}$$

The functions $G^q(H^q)$ read

$$\begin{aligned}
G(z_+,z_-,\kappa_+xp_-,\kappa_-^2x^2) &= \int_0^1 \frac{d\lambda}{\lambda^2} e^{i\kappa_+\lambda xp_-} \\
&\times g\Big(\frac{z_+}{\lambda},\frac{z_-}{\lambda},\kappa_-^2\lambda^2x^2\Big)\theta(\lambda-|z_+|)\theta(\lambda-|z_-|),
\end{aligned}$$

$$\begin{aligned}
\overset{o}{G} &= \partial_{i\kappa_+xp_-}G(z_+,z_-,\kappa_+xp_-,\kappa_-^2x^2) \\
&= \int_0^1 \frac{d\lambda}{\lambda} g\Big(\frac{z_+}{\lambda},\frac{z_-}{\lambda},\kappa_-^2\lambda^2x^2\Big)e^{i\kappa_+\lambda xp_-} \\
&\quad\times\theta(\lambda-|z_+|)\theta(\lambda-|z_-|), \\
G' &= \partial_{\kappa_-^2x^2}G(z_+,z_-,\kappa_+xp_-,\kappa_-^2x^2) \\
&= \int_0^1 d\lambda\,\partial_{\kappa_-^2\lambda^2x^2}g\Big(z_+\lambda,\frac{z_-}{\lambda}\kappa_-^2\lambda^2x^2\Big) \\
&\quad\times\theta(\lambda-|z_+|)\theta(\lambda-|z_-|)e^{i\kappa_+\lambda xp_-}\ .
\end{aligned}$$

Consequences of this representation are the integral relations in polarized deeply inelastic scattering in the forward case [5]. Analogous decompositions and representations are valid for the matrix elements of the pseudo–scalar and pseudo–vector twist–2 quark operators. The corresponding partition functions are denoted by $(g_5^q,\, h_5^q)$ and $(G_5^q,\, H_5^q)$, respectively, and the Dirac and Pauli structures are to be replaced substituting $\gamma_\mu \to \gamma_5\gamma_\mu$ and $\sigma_{\mu\nu}\to\gamma_5\sigma_{\mu\nu}$. With the prerequisites provided above we now derive the asymptotic representation of the Compton scattering amplitude in the generalized Bjorken region, noting that $\xi = Q^2/qp_+, \eta = qp_-/qp_+$,

$$\begin{aligned}
T^{\mathrm{tw2}}_{\mu\nu}(p_+,p_-,q) = \int Dz\Big\{ \\
&\Big(\frac{1}{\xi+t-i\varepsilon}-\frac{1}{\xi-t-i\varepsilon}\Big)F^{(1)}_{\mu\nu} \\
&+\Big(\frac{1}{(\xi+t-i\varepsilon)^2}+\frac{1}{(\xi-t-i\varepsilon)^2}\Big)F^{(2)}_{\mu\nu} \\
&+\Big(\frac{1}{(\xi+t-i\varepsilon)^3}-\frac{1}{(\xi-t-i\varepsilon)^3}\Big)F^{(3)}_{\mu\nu} \\
&+\Big(\frac{1}{\xi+t-i\varepsilon}+\frac{1}{\xi-t-i\varepsilon}\Big)F^{(1)}_{5,\mu\nu} \\
&+\Big(\frac{1}{(\xi+t-i\varepsilon)^2}-\frac{1}{(\xi-t-i\varepsilon)^2}\Big)F^{(2)}_{5,\mu\nu}\Big\}.
\end{aligned}\tag{6}$$

With $t = z_+ + \eta z_-$ Eq. (6) is the two–variable representation of the nonforward Compton amplitude. One may generalize Eq. (6) accounting for current conservation explicitly. The functions $F^{(k)}_{(5)\mu\nu}(p_+,p_-,q;z_+,z_-)$ are (anti)symmetric w.r.t. the Lorentz indices and contain the functions G, H, momentum vectors and the spinor structures $\bar{u}\gamma^\mu u$ etc. As an exam-

ple we present one of these functions here [1]:

$$F^{(1)}_{\mu\nu} = \frac{1}{qp_+}\left[q^\alpha g_{\mu\nu} - (g^\alpha_{\ \nu} q_\mu + g^\alpha_{\ \mu} q_\nu)\right] \times [\bar{u}(p_2)\gamma_\alpha u(p_1) G^q + \Big(\bar{u}(p_2)(\sigma p_-)^\alpha u(p_1)/M\Big) H^q]. \quad (7)$$

It is, however, also possible to derive the one–variable representation from Eq. (6). In this case we use instead of the variables z_+, z_- the variables t, z_-. Then the integrations factorize. The t–integration remains as overall integration and the z_-–integration has to be performed in the functions $F^{(i)}_{(5),\mu\nu}$ which depend on the functions G, H. Finally the z_-–integration acts on the functions $G^q_{(5)}(H^q_{(5)})$ only, leading to new functions

$$\widehat{F}^q_{(5)}(t,\eta) = \int_{-1}^{+1} dz_- F^q_{(5)}(z_+ = t - \eta z_-, z_-) \times\theta(1 + t - \eta z_- + z_-)\theta(1 + t - \eta z_- - z_-) \times\theta(1 - t + \eta z_- + z_-)\theta(1 - t + \eta z_- - z_-),$$

with $F = G$ or H, which depend on the variables t and η. In the limit of forward scattering, $p_- \to 0$, the Compton amplitude does not depend on the distribution amplitudes $H^q_{(5)}(z_+, z_-)$ any longer. Using the normalizations $\bar{u}(p)u(p) = 2M, \bar{u}(p)\gamma_\mu u(p) = 2p_\mu$ and $\bar{u}(p)\gamma_5\gamma_\beta u(p) = -2S_\beta, S^2 = -M^2$, we obtain

$$T^{\rm sym}_{\mu\nu} = \left(g^{\mu\nu} - \frac{p^\mu q^\nu + p^\nu q^\mu}{pq}\right)\int_{-1}^{1} dz_+ \times\left(\frac{1}{\xi + z_+ - i\varepsilon} - \frac{1}{\xi - z_+ - i\varepsilon}\right)\widehat{g}^q(z_+) + 2q_\mu q_\nu \int Dz G'^q(z_+, z_-) \times\left(\frac{1}{(\xi + z_+ - i\varepsilon)^3} - \frac{1}{(\xi - z_+ - i\varepsilon)^3}\right), \quad (8)$$

$$T^{\rm antisym}_{\mu\nu} = i\varepsilon_{\mu\nu}{}^{\gamma\beta}\frac{q_\gamma p_\beta}{(pq)^2} qS \int_{-1}^{+1} dz_+ \times\left[\frac{1}{\xi + z_+ - i\varepsilon} + \frac{1}{\xi - z_+ - i\varepsilon}\right] \times\left[\int_{z_+}^{\epsilon(z_+)} \frac{dz}{z}\widehat{g}^q_5(z) - \widehat{g}^q_5(z_+)\right] - i\varepsilon_{\mu\nu}{}^{\gamma\beta}\frac{q_\gamma S_\beta}{(pq)}\int_{-1}^{+1} dz_+ \times\left[\frac{1}{\xi + z_+ - i\varepsilon} + \frac{1}{\xi - z_+ - i\varepsilon}\right]\int_{z_+}^{\epsilon(z_+)} \frac{dz}{z}\widehat{g}^q_5(z). \quad (9)$$

Here, the z_-–integral was performed

$$\int_{-1+|z_+|}^{+1-|z_+|} dz_- G^q(z_+, z_-) = \int_{z_+}^{\epsilon(z_+)} \frac{dz'}{z'}\widehat{g}^q(z'),$$

with $\epsilon(z) = \mathrm{sign}(z)$ and

$$\widehat{g}^q(z_+) = \int_{-1-|z_+|}^{1-|z_+|} dz_- g^q(z_+, z_-).$$

From the absorptive part of the Compton amplitude in the forward direction one derives directly the Wandzura–Wilczek relation [5]. Moreover the tensor structure of Eq. (8) implies the Callan–Gross relation. Similarly the other twist–2 relations are obtained [5]. Note that the partition functions G, H and G_5, H_5 are directly related to the partition functions g, h and g_5, h_5 which are defined by the matrix elements of scalar and pseudo–scalar twist–2 quark operators, Eq. (3). This demonstrates again that for the description of virtual Compton scattering the properties of the scalar operators are sufficient in leading order.

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